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ABSTRACT

The pressure variations in the Krafla magma reservoir during inflations and deflations are estimated. An inflation of 45 cm requires an increase of magma pressure of about 10 MPa (100 bar). Following large deflations, the pressure in the magma reservoir is significantly below the lithostatic pressure, but immediately before a major deflation, the magma pressure exceeds the lithostatic pressure by about 10 MPa. The volume of the magma reservoir is found to be about 42 ± 14 km^3, based on the Mogi equation. This is the estimated volume of a spherical body where pressure is equalized. The volume of molten magma within the magma reservoir is estimated between 7 and 47 km^3 from gravity observations and estimated values of the elastic constants.

NOTATION

a Radius of spherical volume of varying pressure.

g Acceleration of gravity.

H Depth to the center of spherical volume of radius $\underline{a}$, or depth to the center of a magma reservoir.

k_l Bulk modulus of magma.

k_s Bulk modulus of solid rock.

P Pressure variation inside spherical volume of radius $\underline{a}$.

r Radial distance from center of spherical volume of radius a.

R Horizontal distance from center of spherical volume of radius $\underline{a}$, or from the center of a magma reservoir.

Volume of sphere of radius $\underline{a}$.

Volume of magma (molten rock) within the sphere of radius $\underline{a}$.

Z Vertical distance from center of spherical volume of radius $\underline{a}$, or from the center of a magma reservoir.

Correction factor, explained in text.

Δh Vertical displacement of the surface.

ΔR Horizontal displacement of the surface.

Δs Particle displacement within the elastic halfspace.

Particle displacement at the boundary of spherical volume of radius a.

ΔV_1 Volume of the inflation bulge/deflation bowl.

ΔV_2 Volume change of the magma reservoir.

Volume of magma which if introduced into the magma reservoir will cause inflation bulge/deflation bowl of volume ΔV_1.

ρ Density of crustal rock.

μ Rigidity of crustal rock.

INTRODUCTION

Monitoring of the ground deformation in the Krafla region since 1976 shows a succession of slow inflations followed by rapid deflations. The inflation bulge has a form which closely resembles that predicted by Mogi (1958) for a small spherical chamber of increasing pressure within a homogeneous elastic half space. The deflation bowl also agrees with the Mogi equation, except in those deflations which are associated with surface rifting through the central part of the Krafla volcanic complex (Björnsson et al., 1979; Johnson et al., 1980; Tryggvason, 1980).

The model which is applied in this paper is based on the observation and can be described briefly as follows:

A spherical chamber of radius $\underline{a}$ lies beneath the central part of the suggested Krafla caldera (Fig. 1), centered at a depth H. The chamber is surrounded by elastic material of rigidity μ and the chamber is partly filled by molten magma of bulk modules $\underline{k_1}$. A pressure increase $\underline{P}$ within this chamber will cause vertical displacement Δh of the ground surface at radial distance $\underline{R}$ from surface point vertically above the center of the chamber as given by the "Mogi equation":

$$\Delta h = \frac{3a^3 P}{4\mu} \frac{\underline{H}}{(\underline{H}^2 + \underline{R}^2)^{3/2}} = \Delta h_o \frac{\underline{H}^3}{(\underline{H}^2 + \underline{R}^2)^{3/2}} \tag{1}$$

The horizontal displacement $\underline{\Delta R}$ is similarily given as:

$$\Delta R \quad \frac{3a^3 P}{4\mu} \frac{}{(\underline{H}^2 - \underline{R}^2)^{3/2}} = \underline{\Delta h_o} \frac{\underline{H}^2 \underline{R}}{(\underline{H}^2 - \underline{R}^2)^{3/2}} \tag{2}$$

where Δh_o is the vertical surface displacement vertically above the center of the spherical chamber.

The equations (1) and (2) are claimed to be correct only if $\underline{a} \ll \underline{H}$, but observations in the Krafla area (Björnsson et al., 1979; Tryggvason, 1980; Kjartansson, pers.comm.) show that the shape of the inflation bulge and

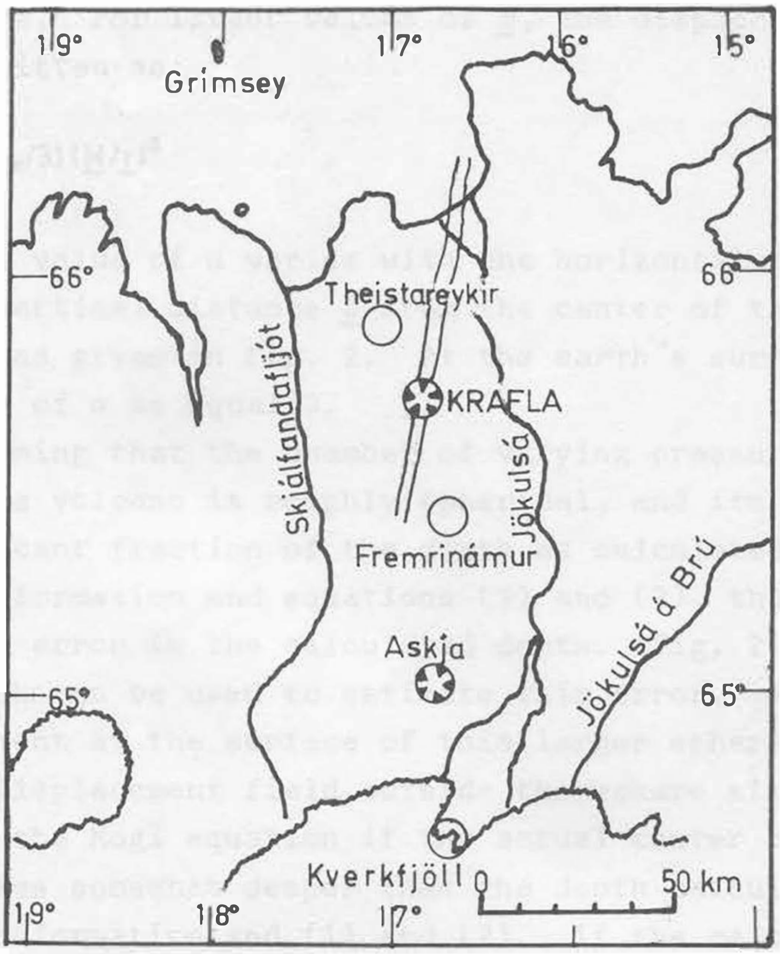

Fig. 1. Location of central volcanoes in the neovolcanic zone of northern Iceland (circles) and the Krafla fissure swarm. Krafla and Askja have developed calderas.

horizontal to vertical displacement ratios agree quite well with these equations, although the presumed magma chamber must be quite large.

The particle displacement Δs within the elastic half space can be obtained from the equations given by Mogi (1958). In the immediate vicinity of the small spherical volume of varying pressure, this particle displacement is nearly radial and its magnitude is approximately

$$\Delta s = (\Delta h_o/3)(H/r)^2 \tag{3}$$

where $r = R^2+Z^2$ is the radial distance from the center of the sphere. For larger values of r, the displacement Δs can be written as:

$$\Delta s = \alpha(\Delta h_o/3)(H/r)^2 \tag{4}$$

where the value of α varies with the horizontal distance R and the vertical distance Z from the center of the spherical body as given in Fig. 2. At the earth´s surface (Z=H) the value of α is equal 3.

Assuming that the chamber of varying pressure beneath the Krafla volcano is roughly spherical, and its radius is a significant fraction of the depth as calculated from ground deformation and equations (1) and (2), then we can expect an error in the calculated depth. Fig. 2 contains data which can be used to estimate this error. A radial displacement at the surface of this larger sphere will produce displacement field outside the sphere similar as given by the Mogi equation if the actual center of the sphere lies somewhat deeper than the depth calculated from surface deformation and (1) and (2). If the calculated depth is H and the actual depth to the center of the sphere is Ha, a radius of the sphere equal to 0.5 H will give Ha about 1.1 H. If the radius of the sphere is 0.7 H, the depth Ha is roughly 1.2 H. Thus a large spherical chamber of varying pressure may cause very similar surface deformation as a much smaller chamber centered at somawhat shallower depth.

The volume changes associated with the inflation and deflation of the Krafla volcano can be estimated and evaluated along several different lines. The volume of the inflation bulge or deflation bowl ΔV_1 can be calculated from observed ground deformation using equations (1):

$$\Delta V_1 = \int_0^\infty 2\pi R\, \Delta h\, dR = 2\pi H^2 \Delta h_o \tag{5}$$

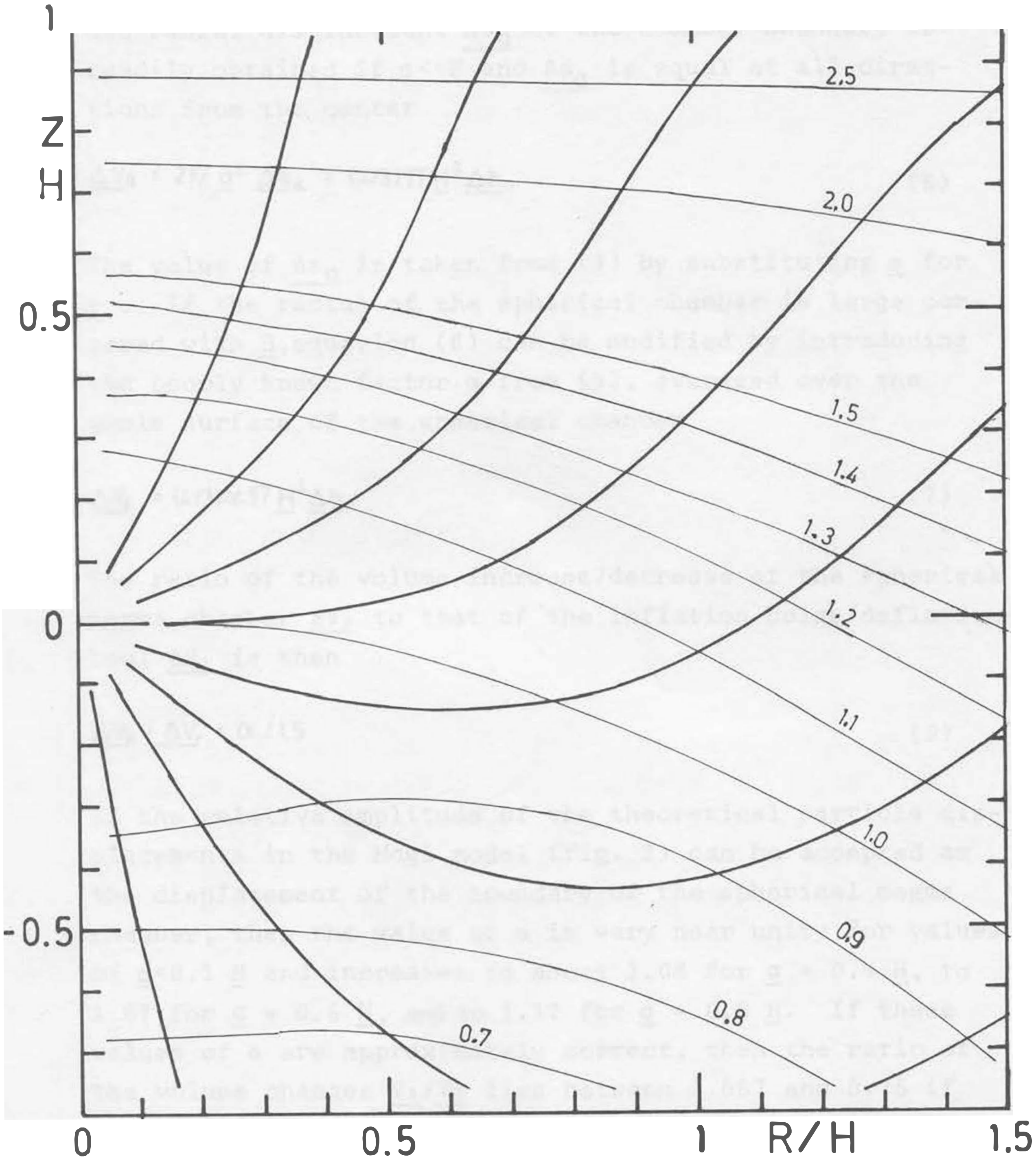

Fig. 2. Direction of particle displacements (thick lines) in elastic half space around a small sphere of varying pressure centered at (0.0). Thin lines are drawn through points of equal value of α, the ratio defined in equations (3) and (4). (Based on equations given by Mogi (1958)).

The volume increase ΔV_2 of the hypothetical spherical chamber of increased or decreased pressure of radius $\underline{a}$, and radial displacement $\underline{\Delta s_a}$ at the chamber boundary is readily obtained if $\underline{a \ll H}$ and $\underline{\Delta s_a}$ is equal at all directions from the center

$$\underline{\Delta V_2} = 2\pi \underline{a}^2 \underline{\Delta s_a} = (4/3)\pi \underline{H}^2 \underline{\Delta h_o} \qquad (6)$$

The value of Δs_a is taken from (3) by substituting $\underline{a}$ for r . If the radius of the spherical chamber is large compared with H, equation (6) can be modified by introducing the poorly known factor α from (4), averaged over the whole surface of the spherical chamber

$$\underline{\Delta V_2} = (4/3)\alpha\pi \underline{H}^2 \underline{\Delta h_o} \qquad (7)$$

The ratio of the volume increase/decrease of the spherical magma chamber $\underline{\Delta V_2}$ to that of the inflation bulge/deflation bowl ΔV_1 is then

$$\underline{\Delta V_2} / \underline{\Delta V_1} = \alpha / 1.5 \qquad (8)$$

If the relative amplitude of the theoretical particle displacements in the Mogi model (Fig. 2) can be accepted as the displacement of the boundary of the spherical magma chamber, then the value of α is very near unity for values of $a < 0.1$ H and increases to about 1.03 for $a = 0.4$ $\underline{H}$, to 1.07 for $a = 0.5$ $\underline{H}$, and to 1.12 for $\underline{a} = 0.6$ H. If these values of α are approximately correct, then the ratio of the volume changes $\underline{V_2}/\underline{V_1}$ lies between 0.667 and 0.75 if the radius of the hypothetical magma chamber is less than 0.6 H. For still larger magma chamber, this ratio will approach unity.

A third volume change, $\underline{\Delta V_3}$ can be defined as the volume of magma at constant pressure which, if introduced into the magma chamber, will cause a volume change ΔV_2 of the magma chamber.

If the volume $\underline{V_1} = 4/3\pi \underline{a}^3$, that of the spherical chamber of varying pressure contains the volume $\underline{V_2}$ of liquid magma of bulk modules k_l and the volume $\underline{V_1}-\underline{V_2}$ of solid rock (and crystals) of bulk modules k_s, then the volume $\underline{\Delta V_3}$ of added (or withdrawn) magma is given by:

$$\underline{\Delta V_3} = \underline{\Delta V_2} + \underline{V_2}\,\underline{P}/\underline{k_l} + (\underline{V_1}-\underline{V_2})\underline{P}/\underline{k_s} \tag{9}$$

As the bulk modulus of solid rock (k_s) is much greater than that of liquid magma (k_l) the last term of (9) can be deleted without introducing major error (Blake, 1981), especially if most of the material inside the sphere of radius $\underline{a}$ is molten ($V_1 > V_2 > 0.5\ \underline{V_1}$).

With this modification (9) can be rewritten as:

$$\underline{\Delta V_3} = \underline{\Delta V_2} + (8/9)\,\underline{\Delta V_1}\,(\mu/\underline{k_l})\,(\underline{V_2}/\underline{V_1}) \tag{10}$$

The value of μ, the rigidity of crustal rock, can be obtained from observed seismic velocities. For the Krafla region this is approximately 17 GPa according to velocities reported by Palmason (1971). The value of k_l, the bulk modulus of fully molten magma is given as 14 GPa by Blake (1981) for basaltic composition of magma. The value of $\underline{\Delta V_2}$, the volume change of the spherical magma chamber can be estimated crudely as $0.67\ \alpha\ \underline{\Delta V_1}$ according to the above discussion. Then (10) can be written once more as:

$$\underline{\Delta V_3} = \underline{\Delta V_1}\,(0.67\alpha + 1.08\,\underline{V_2}/\underline{V_1}) \tag{11}$$

The value of $\underline{\Delta V_1}$ is obtained from measured ground deformation and equation (1) and (5). The value of $\underline{V_1}$ can be calculated from (1) if the relation between $\underline{P}$ and $\underline{\Delta V_1}$ is known. The value of α can be estimated crudely from Fig. 2 and the ratio $\underline{\Delta V_3/\Delta V_1}$ can be obtained from gravity observations together with deformation measurements, which show the relation between mass changes and volume changes.

PRESSURE IN THE KRAFLA MAGMA RESERVOIR

Each subsidence event of the Krafla volcano commences rather suddenly and the rate of subsidence usually reaches a maximum within a few hours. Thereafter the rate of subsidence decreases nearly exponentionally (Tryggvason, 1980). The roughly exponentional decrease of the subsidence rate indicates that the driving force approaches zero towards the end of each subsidence event.

During each subsidence event, magma flows from the Krafla magma reservoir along the Krafla fissure swarm, to be deposited in fissure or as lava on the earth´s surface. Open surface fissures and widening of the fissure swarm are observed in the area of magma deposition (Björnsson et al., 1979). The presence of new open fissures show that the pressure of the new lava within the fissure swarm is less than the lithostatic pressure in cases of no eruption, but in cases of eruptions, the direct connection between the Krafla magma reservoir and the eruptive vent is assumed to mean, that at the end of each eruption, the elevation of the eruption site and the density of the magma controls the pressure of the whole interconnected magma body.

In case of no eruption, the elevation of the part of the fissure swarm, where the magma is deposited, gives a value of maximum pressure in the magma system at the end of the subsidence event. The pressure in the Krafla magma chamber at the end of each subsidence event is then less than lithostatic pressure by an amount controlled by land elevation difference above the magma reservoir (about 550 m) and above the deposited magma (base elevation).

It is argued above, that the "Mogi equation" is approximately valid for the Krafla deformation. This implies that pressure variations in the magma reservoir are proportional to the deflations/inflations.

If we further assume, that the excess pressure needed to start a deflation (rupture strength) is always the same, there should be a near linear relation between the amount of subsidence and the lowest land elevation above the deposited magma, provided open fissures were formed at that location.

As pressure increase during inflation of the Krafla volcano also is proportional to the amount of inflation, similar linear relation should exist between the land elevation above the deposited magma and the amount of inflation from end of one subsidence event to the beginning of next event.

Figs. 3 and 4 show this relation between inflation/deflation and land elevation above deposited magma. In case of eruptions, which last as long as the deflation, the elevation of the last eruption vent is accepted as base elevation. Minor subsidence events with no observed surface fissures are included in the figures (3 and 4), but not used to determine the linear fit. These minor events are considered as incomplete in the sense that they were terminated before the pressure in the magma reservoir was reduced to the value controlled by land elevation above the deposited magma. The reason for this may be the strength of the upper crustal rocks in the region of deposition, which were not exceeded by magma pressure.

The slope of the best linear fit between deflation/inflation and land elevation above deposited magma is about 36 m (32.8 m for deflations, 38.5 m for inflations) in land elevation per 1 mm on the water tube tiltmeter in the Krafla power house or 5 cm inflation/deflation above the center of the magma reservoir.

The linear fit of the relation between inflation and land elevation is significantly better (r^2 = 0.88, Fig. 4), than that between deflation and land elevation (r^2 = 0.84, Fig. 3).

The pressure difference corresponding to 36 m of land elevation is $P = 36 \times \rho \times g$ where ρ is the average rock density about 2700 kg/m^3 and g is the acceleration of gravity, about 9.82 m/sec^2. This gives $P \simeq 1$ MPa for 5 cm inflation/deflation above the magma reservoir.

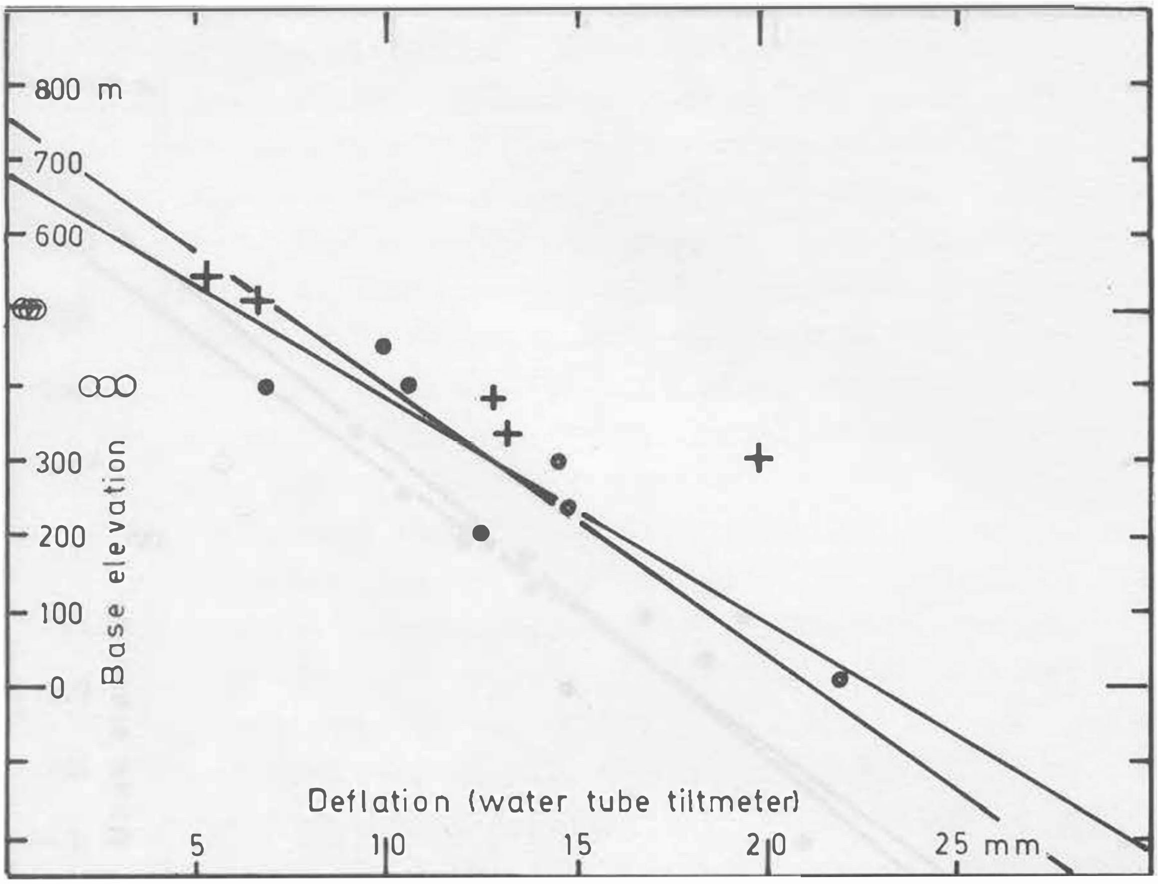

Fig. 3. North component of tilt in the Krafla power house during deflation (subsidence) events plotted against the lowest land elevation of fractured ground in the Krafla fissure swarm during the same event (base elevation). The readings of a 68.95 m long water tube tiltmeter are used for the tilt, but ground displacements in the central part of the deflation bowl are about 50 times greater than the tiltmeter readings (Björnsson et al., 1979). Small circles denote large deflation events with no observed faultings in the central part of the deflation area. Only these are used to determine the two regression lines shown. Crosses denote large deflation events with more or less extensive faulting through the deflation area. Large open circles denote minor subsidence events of poorly determined base elevation.

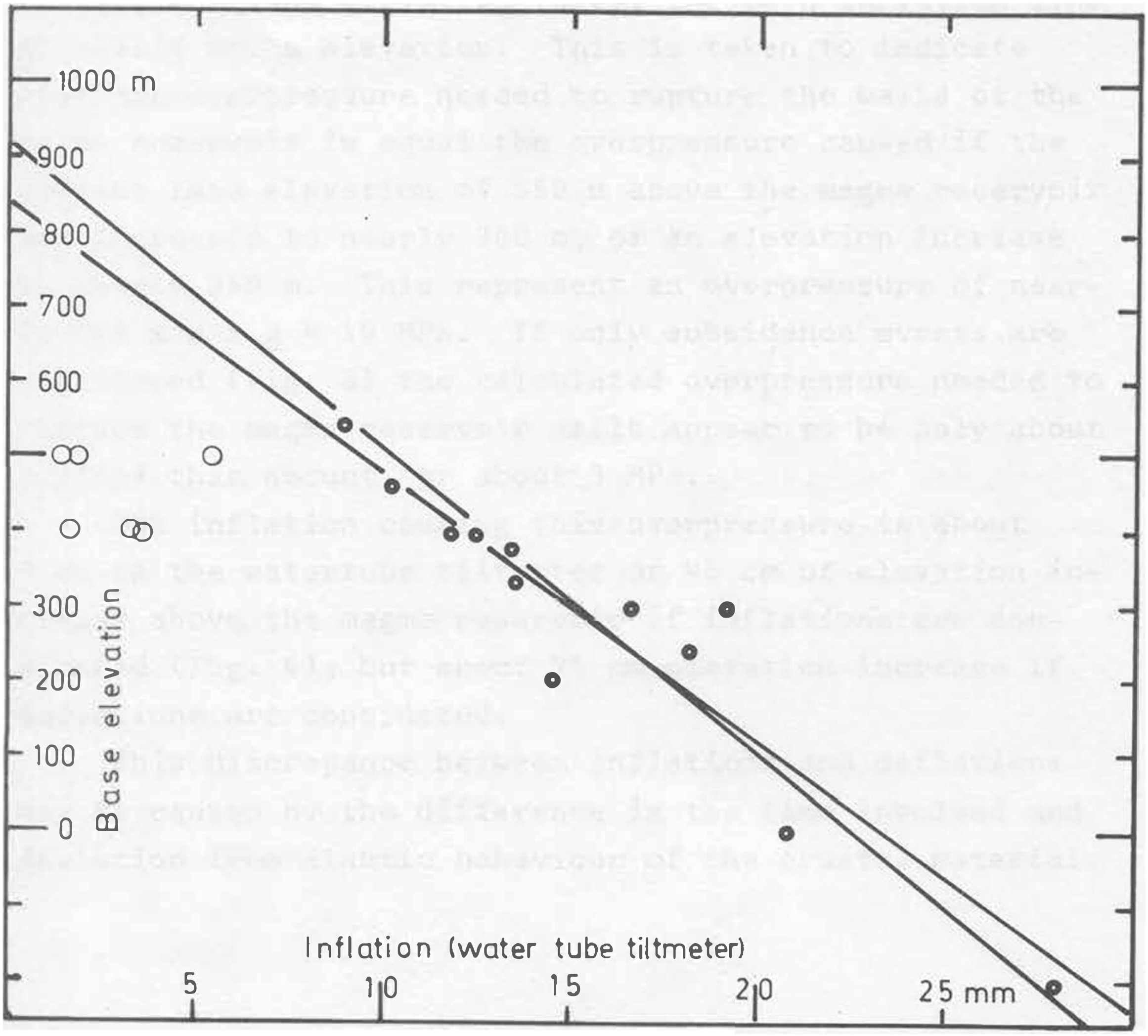

Fig. 4. South component of tilt in the Krafla power house during inflation periods plotted against base elevation of preceeding deflation event. Small circles denote inflations after large (>5 mm) subsidence events until next large subsidence events. They are used to calculate the regression lines shown. Large open circles denote inflation after small (<5 mm) subsidence events until the following large subsidence event.

If only inflations are considered (Fig. 4) we get the intercept between the best line through observed inflation and land elevation to cut the zero inflation line at nearly 900 m elevation. This is taken to indicate that the overpressure needed to rupture the walls of the magma reservoir is equal the overpressure caused if the present land elevation of 550 m above the magma reservoir was increased to nearly 900 m, or an elevation increase by nearly 350 m. This represent an overpressure of nearly $350 \times \rho \times \underline{g} \simeq 10$ MPa. If only subsidence events are considered (Fig. 3) the calculated overpressure needed to rupture the magma reservoir walls appear to be only about half of this amount, or about 5 MPa.

The inflation causing this overpressure is about 9 mm on the watertube tiltmeter or 45 cm of elevation increase above the magma reservoir if inflations are considered (Fig. 4), but about 25 cm elevation increase if deflations are considered.

This discrepance between inflations and deflations may be caused by the difference in the time involved and deviation from elastic behaviour of the crustal material.

VOLUME OF THE MAGMA RESERVOIR AT KRAFLA

The above discussion shows that an increase of 1 MPa pressure in the magma reservoir produces an inflation of 5 (±1) cm above the center of the reservoir. The inflation bulge has a shape which agrees with the Mogi model (equation 1) if the center of the spherical volume of increased pressure lies at 3.0 km depth. This gives the volume ΔV_1 of the inflation bulge for 1 MPa increase of magma pressure as (5):

$$\underline{\Delta V_1} = 2.83 \; 10^6 \; m^3 \qquad (12)$$

The rigidity μ of crustal rock is estimated above as 17 GPa. Incerting these values into (1) for R = 0 we get:

$$a = 2170 \text{ m} \tag{13}$$

and the volume V_1 of the sphere where pressure varies is then:

$$V_1 = (4/3)\pi a^3 = 4.23\ 10^{10}\ \text{m}^3 = 42.3\ \text{km}^3 \tag{14}$$

As the radius of the "spherical" magma reservoir is large compared with its depth, the validity of the "Mogi model" is in serious doubt. The discussion above indicates that the center of the magma reservoir lies deeper than that calculated from (1) by a factor of about 1.2. This means that the actual center of the magma reservoir lies at a depth of roughly 3600 m.

The volume increase of the "spherical magma chamber ΔV_2 is obtained from (8) by estimating the value of α as about 1.25($\pm$0.1):

$$\Delta V_2 = 0.83\ \underline{\Delta V_1} = 2.36\ 10^6\ \text{m}^3 \tag{15}$$

for 5 cm inflation above the center of the magma reservoir.

The volume $\underline{\Delta V_3}$ of magma which is introduced into the magma chamber to give the above inflation is found from (11) by assuming α = 1.25:

$$\underline{\Delta V_3} \simeq \underline{\Delta V_1}(0.83 + 1.08\ \underline{V_2}/\underline{V_1}) \tag{16}$$

where $\underline{V_2}$ is the volume of liquid magma, less or equal $\underline{V_1}$ the volume of the "spherical" magma reservoir.

Johnsen et al. (1980) compared the added or withdrawn mass as estimated from gravity observations, and added or decreased volume of the inflation bulge for two deflation events and one inflation period of the Krafla volcano.

For the two deflations, the average ratio of mass withdrawal to volume decrease gave an apparent density of the withdrawn material as 4.7 g/cm³. Assuming the density of the magma as 2.7 g/cm³, then the ratio $\underline{\Delta V_3/\Delta V_1} = 1.75$. Introducing this into (16) we get:

$$\underline{V_2}/\underline{V_1} = 0.85 \quad \text{or} \quad \underline{V_2} = 36 \text{ km}^3 \tag{17}$$

For one inflation period the apparent density was found to be 3.0 g/cm³ which gives:

$$V_2/V_1 = 0.26 \quad \text{or} \quad \underline{V_2} = 11 \text{ km}^3 \tag{18}$$

The great difference in the apparent density of material introduced into or removed from the magma reservoir is unexplained.

The above volume estimates are subject to large possible errors. The estimated volume $\underline{V_1}$ of the presumed sphere where pressure is equalized, is proportional to the pressure variation P needed to increase this volume by a fixed amount. The relative error of P (Fig. 3, 4) can be estimated as 20% which causes the same relative error in V_1. Additional errors are introduced by assuming that the Mogi equation (1) is valid.

The probable error of the estimated volume V_1 is thus greater than $\underline{V_1}/5$, possible as great as $\underline{V_1}/3$, which gives the value of $\underline{V_1}$ as 42±14 km .

The two estimates of the ratio $\underline{V_2}/\underline{V_1}$, of the molten magma to the spherical volume, where pressure is equalized, are 0.85 and 0.26. The low value gives a volume V_2 of molten magma in the Krafla magma reservoir as 7 to 15 km³ while the high value gives V_2 as 24 to 47 km³. The probable value of V_2 thus lies between 7 and 47 km³.

CONCLUSION

The observed ground deformation in the Krafla area can be used to estimate pressure variation in a magma reservoir, and the volume of this reservoir. These estimate are subject to large possible errors, partly because of insufficient accuracy of measurements and partly because of uncertain modelling of the magmatic processes.

The main results of the present study can be summarized as follows:

Pressure in the magma reservoir varies linearily with the inflation stage. A pressure increase of 1 MPa causes the ground above the reservoir to rise 5 ± 1 cm.

Rupture of the magma chamber occurs when the magma pressure has exceeded the lithostatic pressure by about 10 MPa. This is considered the breaking strength of the crustal rock surrounding the magma reservoir.

The volume of the magma reservoir can be estimated from the pressure variations associated with inflation, and the rigidity of the surrounding crustal rock. This estimate gives the volume as 42 ± 14 km^3, and the depth to the center of this reservoir is crudely estimated as 3600 m. This volume can include considerable amount of solid rock.

The volume of molten magma within the magma reservoir is estimated from apparent density of added or withdrawn volume, based on gravity observations and estimates of the bulk modulus. This estimate gives the molten magma as 26 to 85 per cent of the above volume of 42 ± 14 km^3. Thus the volume of molten magma within the Krafla magma reservoir is estimated as between 7 and 47 km^3.

This volume can be compared with that found by Tryggvason by comparing the volume added in the Krafla fissure swarm to the accumulated volume decrease of the inflation bulge (Tryggvason, 1981). His estimate gave the volume of molten magma in the Krafla magma reservoir as 32 km^3 which lies within the limits of the present estimate.

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